Unit 2 Representing Patterns in Multiple Ways

Lesson Outline

BIG PICTURE

Students will:

- represent linear growing patterns (where the terms are whole numbers) using graphs, algebraic expressions, and equations;
- model linear relationships graphically.

Day	Lesson Title	Math Learning Goals Expectations			
1	What Do Patterns Tell Us?	 Review patterning in real contexts, e.g., weather patterns, quilt patterns, patterns of behaviour, patterns in a number sequence or codes. Develop an understanding that all patterns follow some order or rule, and practice verbally expressing patterning rules. 	8m56 CGE 2c, 3e		
2	Different Representations of the Same Patterns	• Examine (linear) patterns involving whole numbers presented in a variety of forms e.g., as a numerical sequence, a graph, a chart, a physical model, in order to develop strategies for identifying patterns.	8m56, 8m57, 8m60, 8m78 CGE 3b, 5a		
3	Finding the <i>n</i> th Term	 Determine and represent algebraically, the general term of a linear pattern (nth term). Determine any term, given its term number, in a linear pattern represented graphically or algebraically. Check validity by substituting values. 	8m57, 8m58, 8m60, 8m62, 8m63, 8m78 CGE 5b, 7j		
4	Exploring Patterns	 Determine any term given its term number in a linear pattern represented algebraically. Examine patterns involving whole numbers in a variety of forms. Explore and establish the difference between linear and non-linear patterns. 	8m57, 8m58, 8m60, 8m63, 8m73 CGE 3c, 4a		
5	Space Race: Graphic Representations	 Record linear sequences using tables of values and graphs. Draw conclusions about linear patterns. 	8m58, 8m63, 8m78 CGE 4f, 5a		
6	When Can I Buy This Bike?	 Solve problems involving patterns. Use multiple representations of the same pattern to help solve problems. Model linear relationships in a variety of ways to solve a problem. 	8m56, 8m57, 8m58, 8m73, 8m60, 8m63, 8m78 CGE 3c, 4f		
7	Determining the Term Number (lesson not included)	 Determine any term, given its term number, in a linear pattern represented graphically or algebraically. Determine the term number given several terms. 	8m58, 8m61 CGE 3c, 4b, 4f		



Math Learning Goals

- Review patterning in real contexts, e.g., weather patterns, quilt patterns, patterns of behaviour, patterns in a number sequence or code.
- Develop an understanding that all patterns follow some order or rule and practice verbally expressing patterning rules.

Materials

- chart paper
- · variety of everyday patterns
- variety of manipulatives
- BLM 2.1.1, 2.1.2

Assessment Opportunities

Students should be in heterogeneous groupings. A recorder can be assigned in each group or all students may be involved in

Encourage multiple representations of patterns.

recording.

Minds On... Small Groups → Graffiti

Based on class size, set up three stations with different patterning examples at each station, e.g., atlases/maps (landforms, weather), artwork, pine cones, nautilus shells, bird migration patterns. Student groups at each station record all the patterns they discover in 1-2 minutes. Student rotate through all three stations.

Student groups summarize their findings and each group presents a brief summary to the class.

Action!

Think/Pair/Share → Demonstration

Using manipulatives, e.g., linking cubes, display the following patterns: 4, 8, 12, 16... and 1, 4, 7, 10.... Students determine a pattern and share with their partner.

In a class discussion students express the pattern in more than one way, e.g., the first pattern increases by 4 each term, or the pattern is 4 times the term number, the pattern is multiples of 4; the second pattern increases by 3 each term, the pattern is 3 times the term number subtract 2.

Individual → Practice

Students complete BLM 2.1.1, extending the pattern and expressing it in words.

Content Expectations/Observation/Journal/Mental Note: Circulate to assess for understanding of representing patterns.





Consolidate Whole Class → Presentation

Students represent the patterns visually and explain them.

Home Activity or Further Classroom Consolidation

Exploration Reflection

Find a pattern that you like. Record the pattern in your math journal in pictures and words.

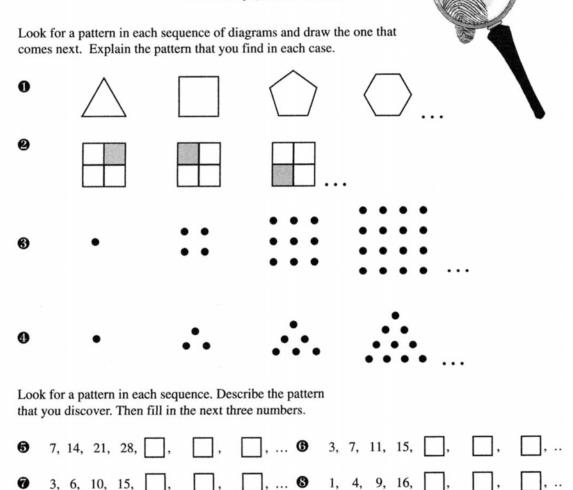
Provide examples of patterns within the class.

2.1.1: Pattern Sleuthing

Pattern Sleuthing

A Mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.

G. H. Hardy (number theorist)



Watch out for this one

1, 1, 2, 3, 5, , , , , , , ,

2, 4, 8, 16,

2, 6, 12, 20,

Write an algebraic expression for the n^{th} term for as many of the sequences from \bullet to \bullet as you can.

, ... **@**

Impact Math: Patterning and Algebra p. 16

3, 8, 15, 24,

4, 6, 10, 18,

Extension

2.1.2: Pattern Sleuthing (Teacher)

Possible student answers:

- 1. Number of sides increases on each polygon, with each term (next shape heptagon)
- 2. Shaded square location rotating counter-clockwise around square pattern (next diagram shaded in lower right area)
- 3. Increasing by odd numbers (3,5,7...) or square numbers (next term 25 dots)
- 4. Adding a row to the bottom of the diagram, with one more dot (next term row added with 5 dots)
- 5. Each term increasing by 7 (35, 42, 49) extension answer: 7*n*
- 6. Each term increasing by 4 (19, 23, 27) extension answer: 4n –1
- 7. Increasing by 3, by 4, by 5, etc. Related to question $3 \text{extension answer: } \frac{n^2 + n}{2} + (n+1)$
- 8. Increasing by consecutive odd numbers (25, 36, 59) extension answer: n^2
- 9. Increasing by consecutive odd numbers (35, 58, 73) extension answer: $n^2 + 2n$ or n(n + 2)
- 10. Each number is doubled (32, 64, 128) extension answer: 2ⁿ
- 11. Increasing by 2, by 4, by 8, by 16 (34, 66, 130) extension answer: 2ⁿ + 2
- 12. Increasing by 4, by, 6, by 8 or by consecutive even numbers (30,42, 56) extension answer: $n^2 + n$ or n(n + 1)

Extension:

This question is the Fibonacci sequence. The pattern is: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987.

See the following websites:

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html http://www.fuzzygalore.biz/articles/fibonacci_seq.shtml http://en.wikipedia.org/wiki/Fibonacci_number



Math Learning Goals

• Examine (linear) patterns involving whole numbers presented in a variety of forms, e.g., as a numerical sequence, a graph, a chart, a physical model, in order to develop strategies for identifying patterns.

Materials

- a visual pattern
- BLM 2.2.1, 2.2.2, 2.2.3
- linking cubes
- rulers

search.

Assessment

Opportunities

Interesting visual patterns can be found by doing an online image

Minds On... Pair/Share → Patterning

Model how to share a visual pattern, e.g., art, nautilus shell, in both words and pictures. Student A shares the pattern in words and pictures with Student B. Student B shares the pattern in words and pictures with Student A. Regroup pairs to form groups of four.

Student A in each pair will share Student B's pattern with the group. Student B in each pair will share Student A's pattern with the group.

Action!

Small Groups → Investigation

In heterogeneous groups, students rotate through the stations (BLM 2.2.1) They record their work on BLM 2.2.2. (The empty circle area on this BLM is used on Day 3.)

Whole Class → Connecting

Students share their findings and record any corrections on their worksheet. They label the four rectangular sections as: Numerical Model, Graphical Model, Patterning Rule, Concrete Model (BLM 2.2.2).

Lead students to the conclusion that all of these representations show the same pattern:

- What do you notice about the table of values and the concrete representation?
- What are the similarities? (i.e., they are all representations of the same pattern)

Curriculum Expectations/Observation/Checklist: Circulate to assess understanding that the representations all show the same pattern.



Consolidate Whole Class → Four Corners

Post charts in the four corners of the room labelled as: Graphical Model, Patterning Rule, Concrete Model, Numerical Model. Below each label, draw a rough diagram to aid visual learners.

Pose the question: For which model did you find it easiest to extend the pattern? Students travel to the corner that represents their answer and discuss why they think that they found that method easier. One person from each corner shares the group's findings.

Home Activity or Further Classroom Consolidation

Complete the practice questions.

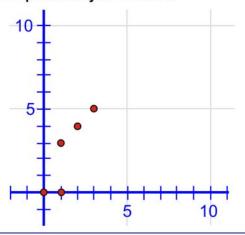
Practice

Provide students with appropriate practice questions showing multiple ways of representing linear patterns.

2.2.1: Stations for Small Group Investigations

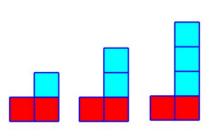
Station 1:

- 1. Examine the graph.
- 2. Plot the next 3 points on your handout.



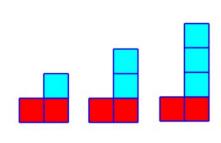
Station 2:

- 1. Using the cubes, build the next 3 models in the pattern.
- 2. On your handout, draw all 6 models.



Station 3:

1. Based on the given models, describe the pattern in words.

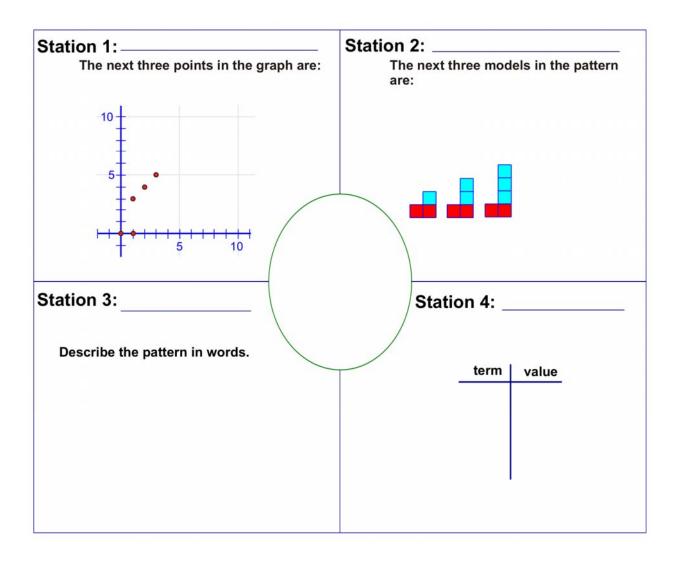


Station 4:

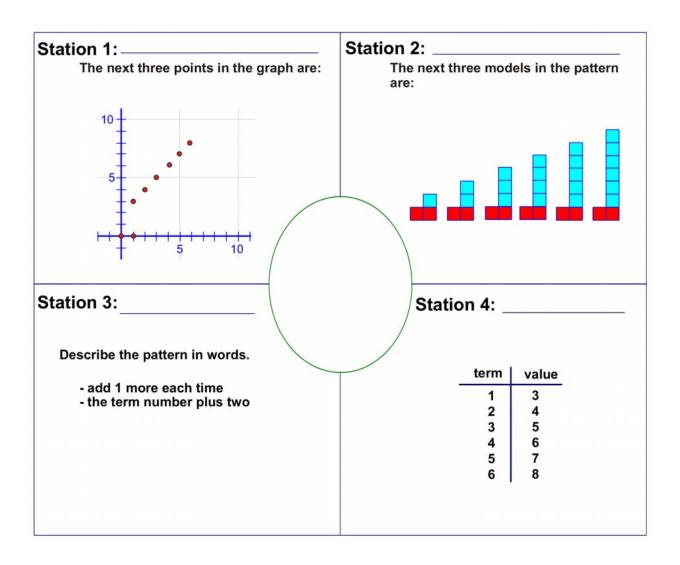
- 1. Draw the table on your handout.
- 2. Complete the table by filling in the blanks.

value	
3	
4	
5	
l .	

2.2.2: Small Group Investigation Record Sheet



2.2.3: Small Group Investigation (Answers)





Math Learning Goals

- Determine, and represent algebraically the general term of a linear pattern $(n^{\text{th}} \text{ term}).$
- Determine any term, given its term number, in a linear pattern represented graphically or algebraically.
- Check validity by substituting values.

Materials

- BLM 2.3.1, 2.3.2, 2.3.3
- · linking cubes

Assessment

Opportunities

Cut BLM 2.3.1 into individual cards.

Collect the cards from students to use in a future activity.

Word Wall term number

term value

Minds On...

Whole Class → Four Corners

Give each student a card. Students travel to the corner that corresponds to the representation on their card, e.g., A student with a card that has a graph goes to the graphical model representation corner. Students discuss "What is challenging about changing from one representation of a pattern to another?" Choose one person from each corner to share the group's conclusions.

Pose the following scenario: Armando has a CD collection. He currently owns 2 CDs. Each week, he purchases a new CD for his collection. How could you represent this in a model? Students in each corner describe the scenario, using the model represented in their corner.

Action!

Small Groups → Investigation

With the class, model the results to the problem using two colours of linking cubes (2 red and 1 green for the first term, 2 red and 2 green for the second term, and so on). Discuss why the first term has 3 CDs in it. Students use linking cubes to build the concrete model of the pattern up to the 6th term and complete BLM 2.3.2 in groups.

Guide a class discussion about students' findings (BLM 2.3.3).

Representing/Oral Questions/Mental Note: Observe students as they work on the small-group activity.



Debrief

Consolidate Whole Class → Algebraic Representation

Ask:

- How can we think about the algebraic expression in another way? Decide what the n^{th} term represents (unknown term; a method to find any term; a "formula").
- How might you find the 12th term of the pattern?
- Is it possible to find the 12th term without extending the table?
- Find the 12th term. Can you use the same method to find the 100th term?
- How can you determine if your nth term is correct? (Substitute the term numbers in for n and the resulting answers should be the term values.) Students record this algebraic representation of the pattern in the circle on the placemat from Day 2 (BLM 2.2.2).

Home Activity or Further Classroom Consolidation

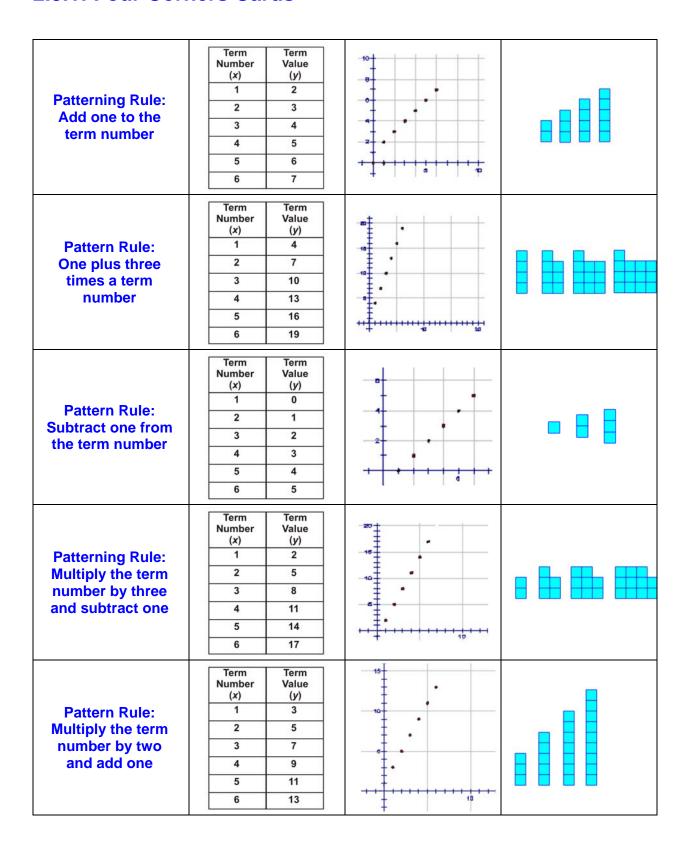
Application Exploration Reflection

Complete the practice questions.

Provide students with appropriate practice questions.

9

2.3.1: Four Corners Cards



2.3.2: Patterns – Finding the nth Term

Term Number (<i>n</i>)	Number of Red Cubes ()	Number of Green Cubes ()	Total Number of Cubes (Term Value)
1			
2			
3			
4			
5			
6			
12			
n			

- 1. In your groups, complete the values for terms 1 through 6 on the chart using models.
- 2. Which colour has the same number of cubes all the way through the chart? This is called the **constant** because it does not change. Indicate this in the brackets under the appropriate heading.
- Which colour has a different number of cubes in each model? This is called the variable because it varies or changes. Please indicate this in the brackets under the appropriate heading.
- 4. How is the **variable** related to the term number?
- 5. In words, describe the pattern.
- 6. If the term number is *n*, how could you figure out how many cubes are in that model?

2.3.3: Patterns – Finding the nth Term Answers (Teacher)

Term Number (<i>n</i>)	Number of Red Cubes (Constant)	Number of Green Cubes (Variable)	Total Number of Cubes (Term Value)
1	2	0	2
2	2	1	3
3	2	2	4
4	2	3	5
5	2	4	6
6	2	5	7
12	2	11	13
n	2	n – 1	2 + n - 1 or $n + 1$ or $1 + n$

- 1. In your groups, complete the values for terms 1 through 6 on the chart using your models.
- 2. Which colour has the same number of cubes all the way through the chart? This is called the **constant** because it does not change. Indicate this in the brackets under the appropriate heading.

There are always the same number of red cubes.

3. Which colour has a different number of cubes in each model? This is called the **variable** because it varies or changes. Please indicate this in the brackets under the appropriate heading.

The number of green cubes changes each term.

4. How is the **variable** related to the term number?

The variable is 1 less than the term number.

5. In words, describe the pattern.

The value is 2 more than 1 less than the term number.

6. If the term number is n, how could you figure out how many cubes are in that model?

$$2 + n - 1$$
 or $n + 1$ or $1 + n$



Math Learning Goals

- Determine any term given its term number in a linear pattern represented algebraically.
- Examine patterns involving whole numbers in a variety of forms.
- Explore and establish the difference between linear and non-linear patterns.

Materials

• BLM, 2.4.1, 2.4.2, 2.4.3

Minds On... Whole Class → Summarizing

Review the terms *constant* and *variable*, using an example from Day 3.

Assessment Opportunities

Think Literacy: Mathematics, Grades 7-9. pp. 40-41

Make available the following materials at each station: linking cubes, geoboards, toothpicks and/or other appropriate materials

Word Wall

- variable
- constant

Action!

Small Groups → Exploration

Students rotate through stations (BLM 2.4.2).

Debrief

Consolidate Whole Class → Summarizing

Discuss the patterns students found during their station work.

Pose questions:

- Which patterns did you find more logical to extend and represent another way?
- Why do you think some were more logical than others?

Create class Frayer models for constant and variable. Formulate a working definition for each term. See BLM 2.4.1.

Define that linear patterns form a straight line that can be shown using a ruler but non-linear patterns do not form a line.

In small groups, students sort the different patterns into two groups: linear and non-linear. Groups justify their sorting to the class.

Curriculum Expectations/Communicating/Observation: Listen as students discuss their choices and justify their reasoning as they sort.

If time permits, demonstrate what linear and nonlinear patterns look like graphically using GSP[®]4. Fathom[™], or spreadsheet software to give meaning to the terms linear and non-linear. Use discrete examples so it is consistent with their work.

Differentiated

Reflection

Home Activity or Further Classroom Consolidation

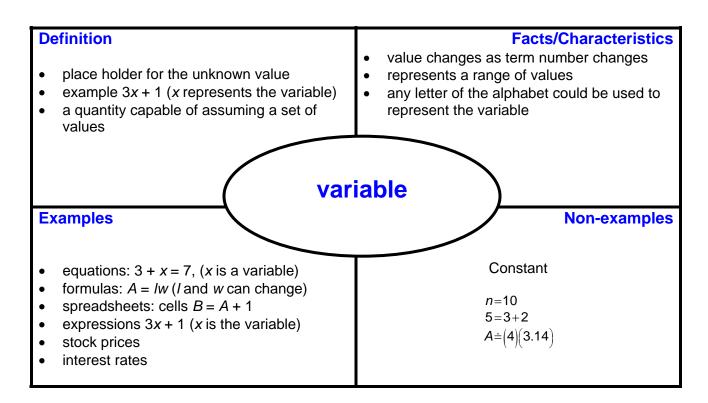
In your journal, compare linear patterns to non-linear patterns, use as many representations as possible.

- How are they similar?
- How are they different?

Make available the GSP®4 take-home version for students who may wish to produce their sketches using software.

2.4.1: The Frayer Model (Teacher)

Definition Facts/Characteristics numerical value that stays the same (is fixed does not change for different terms example: x + 1 (the number 1 is the constant) a quantity that does not change constant **Examples Non-examples** constant pain always the same 5x + 3 (the number 3 is the constant) variable speed of light can represent more than one number n = 1,2,3,45x (the value of the term 5x changes for different values of x)



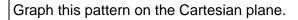
2.4.2: Exploring Patterns

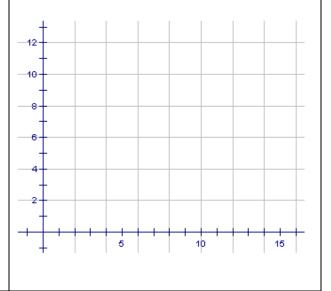
Station 1

(y) 3n – 2
1
4
7
10

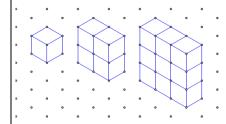
Name the constant.

Name the variable.





Station 2



Create a table of values.

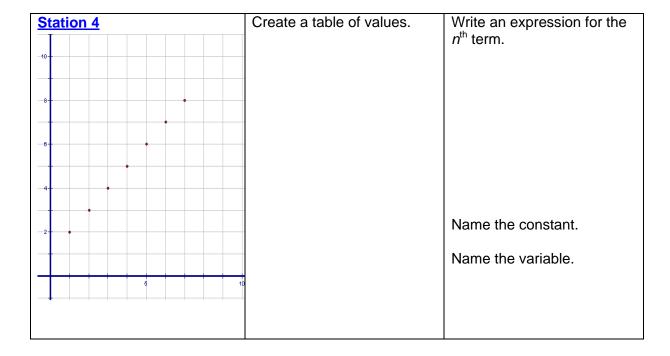
Write an expression for the n^{th} term.

Name the constant.

Name the variable.

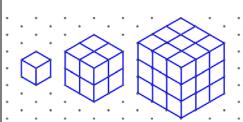
2.4.2: Exploring Patterns (continued)

Station 3	Build the next two terms in the pattern using toothpicks.	Create the table of values using the number of
Note the toothpick pattern below.	Draw them here:	toothpicks.
<u>,-</u> ,		Write an expression for the n^{th} term.
		, term.
		Name the constant.
		Name the variable.



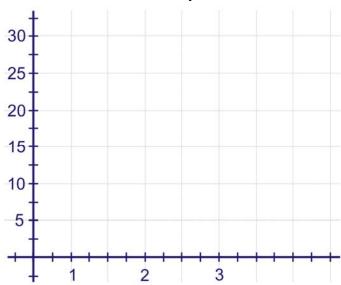
2.4.2: Exploring Patterns (continued)

Station 5



Create a table of values.

Plot the points from your table of values. What do you notice?



Name the constant.

Name the variable.

2.4.2: Exploring Patterns (continued)

Station 6

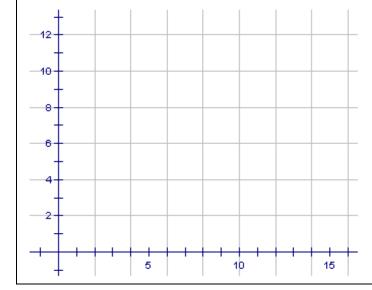
(x)	(y)
3	2
5	3
7	4
9	5

Write an expression for the n^{th} term.

Name the constant.

Name the variable.

Graph this pattern on the Cartesian plane.



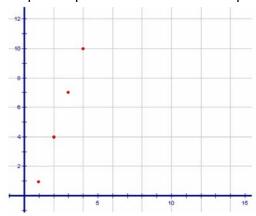
2.4.3: Answers to Student Centres

Station 1

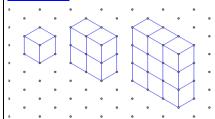
(y) 3n-2
1
4
7
10

Variable: *n*Constant: –2

Graph this pattern on the Cartesian plane.



Station 2



Create a table of values.



Write an expression for the n^{th} term.

$$n^{\text{th}}$$
 term = n^2

Variable: *n* Constant: 0

Station 3

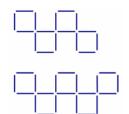
Note the toothpick pattern below.







Build the next two terms in the pattern using toothpicks. Draw them here:



Create the table of values using the number of toothpicks.

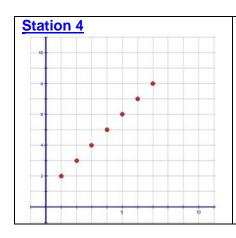
Write an expression for the n^{th} term.

1	4
2	8
3	12
4	16
5	20

 n^{th} term = 4n

Variable: *n* Constant: 0

2.4.3: Answers to Student Centres (continued)



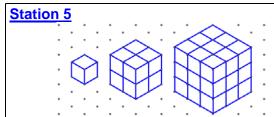
Create a table of values.

1	2
3	4
4	5
7	8

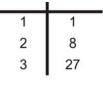
Write an expression for the n^{th} term.

$$n^{\text{th}}$$
 term = $n + 1$

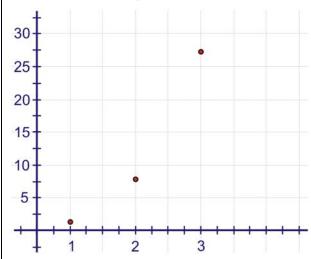
Variable: *n* Constant: 1



Create a table of values.



Plot the points from your table of values. What do you notice?



Variable: *n* Constant: 0

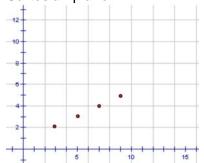
Station 6

(y)
2
3
4
5

Write an expression for the n^{th} term.

$$n^{\text{th}} \text{ term } = \left(\frac{n+1}{2}\right)$$

Variable: nConstant: $\frac{1}{2}$ Graph this pattern on the Cartesian plane.



Assessment Opportunities

Math Learning Goals

Materials





• Record linear sequences using a table of values and graphs.

• Draw conclusions about linear patterns.

Minds On... Whole Class → Connecting to Prior Learning

Create a Venn diagram using the comparison from the Home Activity, Day 4 (BLM 2.5.1).

Action!

Whole Class → Simulation Using Graphs

Using their prior knowledge of linear and non-linear patterns, groups create physical representations of the two types of patterns.

Pose the problem: Using all the people in your group demonstrate what a linear graph would look like.

Observe and comment on how students demonstrate different representations.

Pose a second problem: Using all the people in your group demonstrate what a non-linear graph could look like.

Note how students demonstrate different representations.

Students share their feedback or observations.

Debrief

Consolidate Individual → Interpreting Graphs

Students complete BLM 2.5.2.

Curriculum Expectations/Procedural Knowledge: Students submit BLM 2.5.2 for feedback.



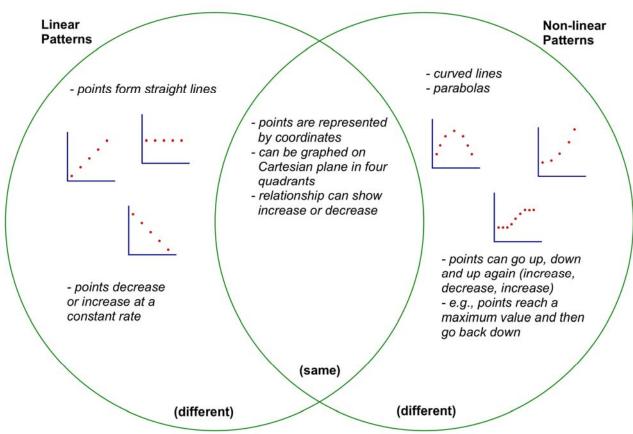
Practice

Home Activity or Further Classroom Consolidation

Complete the practice questions.

Provide students with appropriate practice questions.

2.5.1: Possible Venn Diagram Answers



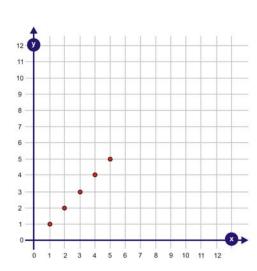
Note: Answers will vary.

2.5.2: Interpreting Graphs

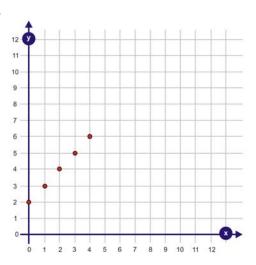
Name:

For each graph below create a table of values and determine the n^{th} term.

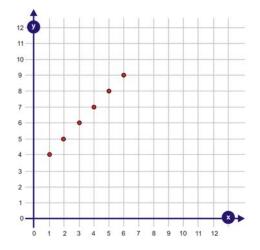
1.



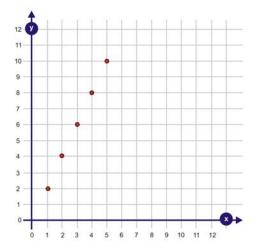
2.



3.

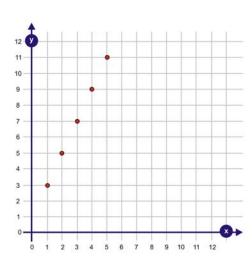


4.

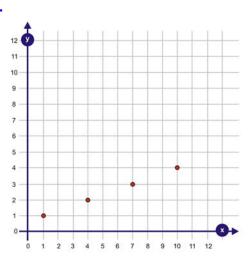


2.5.2: Interpreting Graphs (continued)

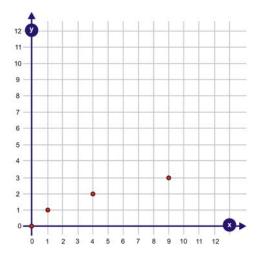
5.



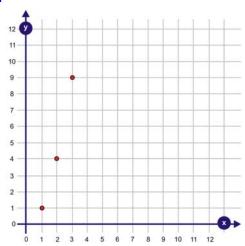
6.



7.



8.





Math Learning Goals

- Solve problems involving patterns.
- Use multiple representations of the same pattern to help solve problems and prove that the solution is correct.
- Model linear relationships in a variety of ways to solve a problem.

Materials

Possible Assumptions:

 bike stays the same price

babysitting money

 she doesn't spend any of the money

stays the same

she saves

• BLM 2.6.1

Assessment Opportunities

Minds On... Whole Class → Review

Hand each student a card (Day 3, BLM 2.3.1). Students find the other members of their group by matching all representations of the same pattern (patterning rule, numerical, graphical, and pictorial).

In their groups, students develop an algebraic expression for their pattern. One student from each group shares the response. (If a group finishes before the others, challenge them to find a story that fits the pattern.)

Action!

Small Groups → Discussion

Explain the task (BLM 2.6.1) and discuss assumptions students must make: What assumptions are you making in order to consider solving this problem?

Students highlight or underline key words in the problem, e.g., costs \$350, received \$300, \$12, per week.

Students use the problem-solving model (understand the problem, make a plan, carry out the plan, look back at the solution) to complete the task and submit their work.

Individual → Performance Task

Students complete this activity using BLM 2.6.1.

Problem Solving/Observation/Checkbric: Circulate to ask probing questions during the performance task.



Think Literacy: Cross Curricular Approaches. Grades 7-12, pp. 76-80, Mind Maps.

Mind maps can be done by hand or with software such as Smart Ideas (Ministry Licensed).

Provide a variety of manipulatives and technology.

Debrief

Consolidate Whole Class → Discussion

Students reflect on the problem-solving model: What strategies did you use for each part of the model? Students share many different strategies and representations.

Home Activity or Further Classroom Consolidation

Reflection

Complete a mind map/web to summarize what you learned in this unit. Use the appropriate vocabulary.

This activity can be used as a review or as an assessment tool.

2.6.1: A Problem-Solving Model: When Can I Buy This Bike?

Name:

Mackenzie has found the bicycle that she always wanted. It costs \$350.00. She received \$300 dollars as a gift from her family. How long would it take her to save enough money to purchase the bike if she earns \$12 a week babysitting?

Using the problem-solving method (Understand the Problem, Make a Plan, Carry out the Plan, Look Back at the Solution) solve the problem above. Explain your thinking using pictures, numbers, and words. You may use manipulatives and a variety of tools to help you determine the solution. If you need more space to show your solution use the back of the page.

Understand the Problem

Read and re-read the problem. Using a highlighter, identify the information given and what needs to be determined.

Write a sentence about what you need to find.

Make a Plan

Consider possible strategies.

Select a strategy or a combination of strategies. Discuss ideas to clarify which strategy or strategies will work best.

Carry Out the Plan

Carry out the strategy, showing words, symbols, diagrams, and calculations. Revise your plan or use a different strategy, if necessary.

Look Back at the Solution

Does your answer make sense? Is there a better way to approach the problem? Describe how you reached the solution and explain it.