

Notes 12.1-12.2 – Arithmetic Sequences & Series – Algebra 2

Learning Target

- I can identify a sequence as Arithmetic or not.
- I can write a rule for the n th term of an Arithmetic sequence.
- I can write a rule given a term and the common difference.

A **sequence** is a list of numbers. We can call a list of numbers an **ARITHMETIC sequence** if you can add or subtract the SAME number each time to get from one to the next. For example, 2,4,6,8... is an ARITHMETIC sequence because I am adding 2 each time.

- Each term in an arithmetic sequence is notated by: a_1 is the first term, a_2 is the second term, and so on.

Example 1:

Find the pattern in the sequence:

$$20, 23, 26, 29, \dots$$

$$\quad +3 \quad +3 \quad +3 \quad +3$$

Adding 3 each time

- Find the 10th term in the list: $a_{10} = 47$
- Find the 100th term in the list: $a_{100} = 317$

If you have an arithmetic sequence, you can use this formula to find numbers in the list:

$$a_n = a_1 + (n - 1)d$$

d is the common difference

a_1 is the first term in the list

n is the place of the term in the list

Example 2: Write a rule for the n th term of the sequence. You need to know the first term and the common difference.

a. 2, 9, 16, 23, ...

$$\quad \checkmark \quad \checkmark \quad \checkmark$$

$$\quad +7 \quad +7 \quad +7$$

$$a_n = 2 + 7(n-1)$$

Use your rule to find the 19th term (a_{19})

$$a_{19} = 2 + 7(19-1)$$

$$a_{19} = 128$$

b. 57, 45, 33, 21, ...

$$\quad \checkmark \quad \checkmark \quad \checkmark$$

$$\quad -12 \quad -12 \quad -12$$

$$a_n = 57 - 12(n-1)$$

Use your rule to find the 50th term (a_{50})

$$a_{50} = 57 - 12(50-1)$$

$$a_{50} = -531$$

Arithmetic Sequence: $a_n = a_1 + (n - 1)d$

Example 3: The eleventh term of an arithmetic sequence is $a_{11} = 41$. The common difference is $d = 5$. Write a rule to find any term in the list.

$$41 = a_1 + 5(11-1) \quad a_n = -9 + 5(n-1)$$

$$41 = a_1 + 50$$

$$-9 = a_1$$

What is the 100th number in the list? $a_{100} = -9 + 5(100-1)$
 $= 486$

Example 4: Two terms of the arithmetic sequence are $a_6 = 7$ and $a_{22} = 87$. Find a rule for the n th term. $a_n = a_1 + d(n-1)$

$$7 = a_1 + d(5) \quad ; \quad 87 = a_1 + d(21)$$

$$7 - 5d = a_1 \quad ; \quad 87 - 21d = a_1$$

$$7 = a_1 + (5)(5) \quad 7 - 5d = 87 - 21d \quad a_n = -18 + 5(n-1)$$

$$7 = a_1 + 25 \quad 16d = 80$$

$$a_1 = -18 \quad d = 5$$

What is the 100th number in the list? $a_{100} = -18 + 5(99)$
 $a_{100} = 477$

ARITHMETIC SERIES means to **ADD** up all the numbers in the arithmetic sequence.

Σ (this is the SIGMA sign. It means to **ADD** up all the terms in the list!)

Example 5: Add up the numbers in the sequence: s_n
 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54 = 495

\rightarrow how many terms you're adding
 $\sum_{i=1}^n (a + di)$ \rightarrow expression you use to evaluate a_1 & a_2
 $i = \#$ \rightarrow the first number in the list

SHORTCUT: Use the formula for arithmetic s $s_n = \frac{n}{2}(a_1 + a_n)$
 n = how many terms you're adding up
 a_1 = the first number in the list
 a_n = the last number in the list

1. $\sum_{i=1}^{18} (77 - 4i)$

$$a_1 = (77 - 4(1)) = 73$$

$$a_{18} = (77 - 4(18)) = 5$$

$$s_{18} = \frac{18}{2} [73 + 5]$$

$$s_{18} = 702$$

2. $\sum_{i=1}^{15} (9 + 3i)$

$$a_1 = (9 + 3(1)) = 12$$

$$a_{15} = (9 + 3(15)) = 54$$

$$s_{15} = \left(\frac{15}{2}\right) (12 + 54)$$

$$s_{15} = 495$$