

Algebra 1
Intro to Slope/Rate of Change

Name: _____
Date: _____ Period: _____

4.3 What is Slope?

If you've ever walked up or down a hill, then you have already experienced a real life example of slope. Keeping this fact in mind, by definition, the slope is the **measure of the steepness of a line**. (In math, slope is defined from left to right)



Positive slope:
If you go from left to right and you go up, the line has a positive slope.

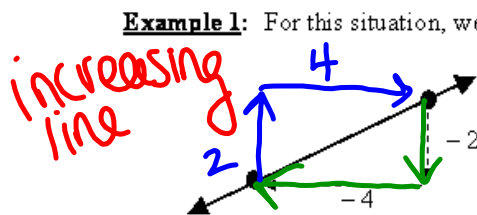
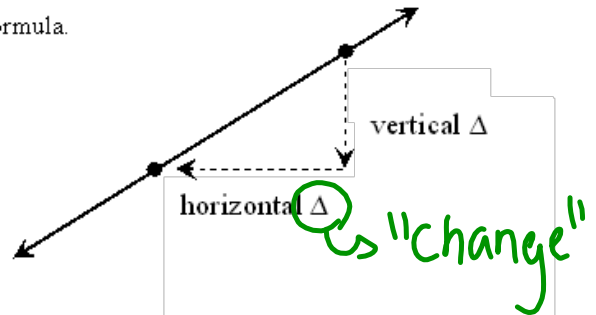
Negative slope:
If you go from left to right and you go down, the line has a negative slope.

Zero slope:
If you go from left to right and you don't go up or down, the line has a zero slope.

Undefined slope:
If you can only go up or you can only go down, the line has an undefined slope.

That steepness can be measured with the following formula.

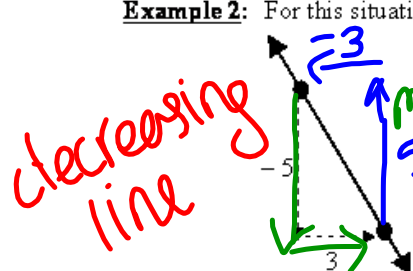
$$\text{slope}(m) = \frac{\text{vertical change}}{\text{horizontal change}}$$



Example 1: For this situation, we see that the vertical change is 2 and the horizontal change is -4.

$$m = \frac{\text{vertical } \Delta}{\text{horizontal } \Delta} = \frac{\downarrow 2}{\leftarrow 4} = \frac{-2}{-4} = \frac{1}{2}$$

$$m = \frac{\uparrow 2}{\rightarrow 4} = \frac{2}{4} = \frac{1}{2}$$



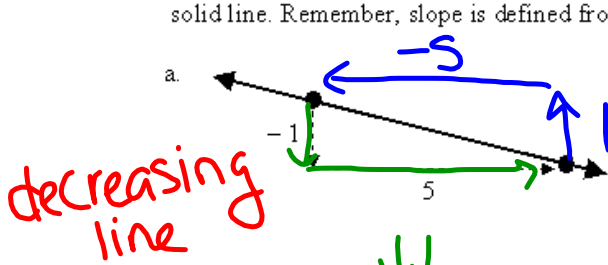
Example 2: For this situation, we see that the vertical change is 5 and the horizontal change is 3.

$$m = \frac{\text{vertical } \Delta}{\text{horizontal } \Delta} = \frac{\downarrow 5}{\rightarrow 3} = \frac{-5}{3}$$

$$m = \frac{\uparrow 5}{\leftarrow 3} = \frac{5}{-3} = \frac{-5}{3}$$

You Try:

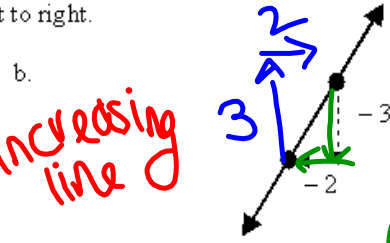
1. Find the vertical change and the horizontal change for each solid line. Then state the slope of the solid line. Remember, slope is defined from left to right.



decreasing line
negative slope

Vertical change = $\frac{\downarrow 1}{\rightarrow 5}$
Horizontal change = $\frac{\rightarrow 5}{\rightarrow 5}$

Slope = $\frac{\downarrow 1}{\rightarrow 5} = \boxed{\frac{-1}{5}}$ $m = \frac{\uparrow 1}{\rightarrow 5} = \frac{1}{5} = -\frac{1}{5} = \boxed{\frac{-1}{5}}$

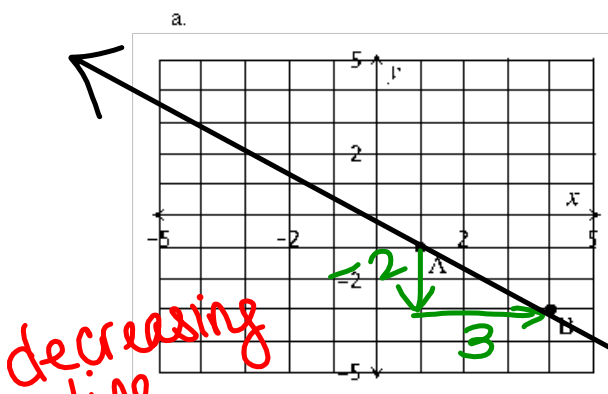


increasing line
positive slope

Vertical change = $\frac{\downarrow 3}{\rightarrow 2}$
Horizontal change = $\frac{\rightarrow 2}{\rightarrow 2}$

Slope = $\frac{\downarrow 3}{\rightarrow 2} = \frac{-3}{2} = \boxed{\frac{3}{2}}$
 $m = \frac{\uparrow 3}{\rightarrow 2} = \frac{3}{2} = \boxed{\frac{3}{2}}$

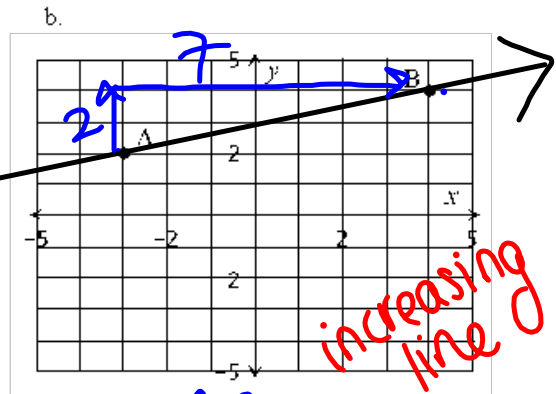
2. Starting at point A find the vertical change and horizontal change to get to point B. Then connect the points to make a solid line. Identify the vertical change, horizontal change, and slope for the line segment between each pair of points below.



decreasing line
negative slope

Vertical $\Delta = \frac{-2}{3}$
Horizontal $\Delta = \frac{\rightarrow 3}{\rightarrow 3}$

Slope = $\boxed{\frac{-2}{3}}$

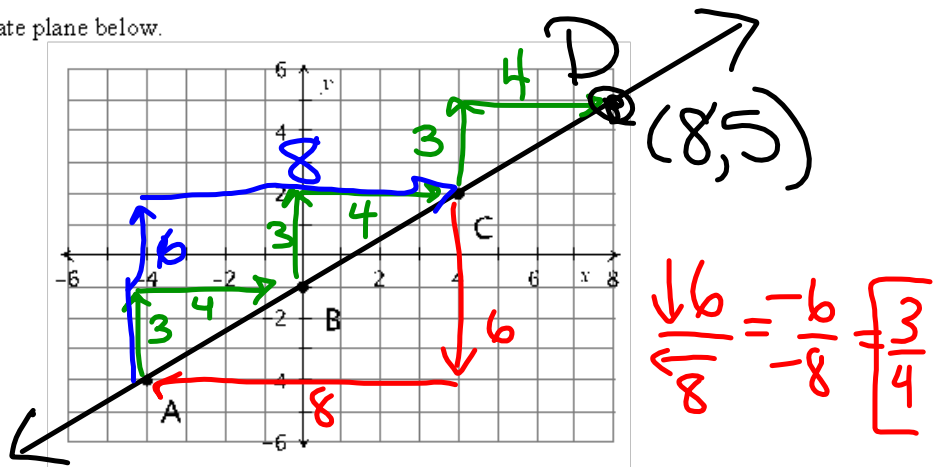


increasing line
positive slope

Vertical $\Delta = \frac{\uparrow 2}{\rightarrow 7}$
Horizontal $\Delta = \frac{\rightarrow 7}{\rightarrow 7}$

Slope = $\frac{\uparrow 2}{\rightarrow 7} = \boxed{\frac{2}{7}}$

3. Use the coordinate plane below.



a. Connect the points using a straightedge. Extend the line past points *A* and *C* and place arrows at each end.

b. Find the slope between points *A* and *B*.
 Vertical $\Delta = \uparrow 3$ Horizontal $\Delta = \rightarrow 4$ Slope = $m = \frac{\uparrow 3}{\rightarrow 4} = \boxed{\frac{3}{4}}$

c. Find the slope between points *B* and *C*.
 Vertical $\Delta = \uparrow 3$ Horizontal $\Delta = \rightarrow 4$ Slope = $m = \frac{\uparrow 3}{\rightarrow 4} = \boxed{\frac{3}{4}}$

d. Find the slope between points *A* and *C*.
 Vertical $\Delta = \uparrow 6$ Horizontal $\Delta = \rightarrow 8$ Slope = $m = \frac{\uparrow 6}{\rightarrow 8} = \frac{6}{8} = \boxed{\frac{3}{4}}$

e. What can you conclude about the slope of this line looking at your results in parts **b** thru **d**?

No matter which point I start from and end at, the slope "comes out to be the same", the slope is always CONSTANT.

f. Starting at point *C* find a fourth point which would belong to the same line. Label your fourth point *D* and explain how you arrived at it using what you know about slope.

$m = \frac{3}{4}$ $\frac{\uparrow 3}{\rightarrow 4}$ Slope is a set of directions I can use to "count" my way to another point on the line

POINT D (8, 5)

4. Now, let's see how to find the slope when we don't know the vertical change and the horizontal change. If we graph the slope on the coordinate system, we will be able to derive another formula for slope using the x and y values of the coordinates.

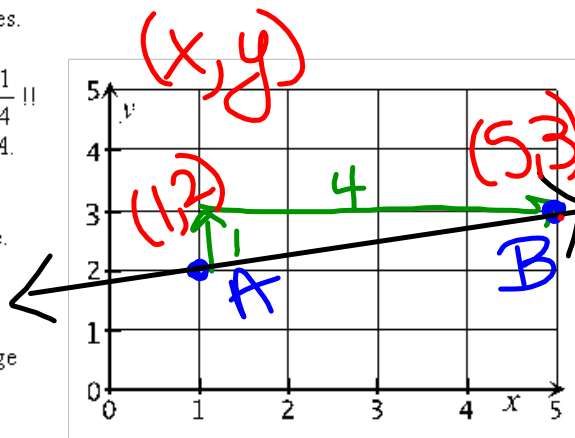
a. Let's draw the graph of a line with a slope of $\frac{1}{4}$!!

Step 1: Plot the point (1, 2) and label it point A.

Step 2: From point A "count your way" to the next point on the line using the given slope. (vertical Δ 1 and horizontal Δ 4). Plot this second point, and label it point B.

Step 3: Connect the points using a straight edge and name the coordinates of point A and point B.

Step 4: Extend the line past points A and B and place arrows at each end.



b. The two coordinates for points A and B can be used to calculate the slope! We should get $\frac{1}{4}$.

A (x_1, y_1)
 $(1, 2)$
 B (x_2, y_2)
 $(5, 3)$

$$m = \frac{\text{vertical } \Delta}{\text{horizontal } \Delta} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{5 - 1} = \boxed{\frac{1}{4}}$$

Does order matter?

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2 - 3}{1 - 5} = \frac{-1}{-4} = \boxed{\frac{1}{4}}$$

Important Definitions for calculating slope:

Vertical Change: difference/change in the y -values ($y_2 - y_1$)

Horizontal Change: difference/change in the x -values ($x_2 - x_1$)

The formula for the slope between the two points A and B can be found by using the x and y coordinates of the two points. Call the ordered pair for point A (x_1, y_1) and the ordered pair for point B (x_2, y_2) .

$$\text{slope} = \frac{\text{vertical } \Delta}{\text{horizontal } \Delta} = \frac{y_2 - y_1}{x_2 - x_1}$$

Formula for slope (m):

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ OR } m = \frac{y_1 - y_2}{x_1 - x_2}$$

5. Use the formula above to find the slope (m) of the line passing through the given points.

a. (1, 5) (2, 9)

$$\begin{array}{l} (1, 5) \\ (2, 9) \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{2 - 1} = \frac{4}{1} = 4$$

b. (2, 4) (1, 1)

will finish these
other examples
tomorrow

c. (4, 0) (8, -2)

d. (-8, 6) (3, 4)

$$\begin{array}{l} (-8, 6) \\ (3, 4) \end{array}$$

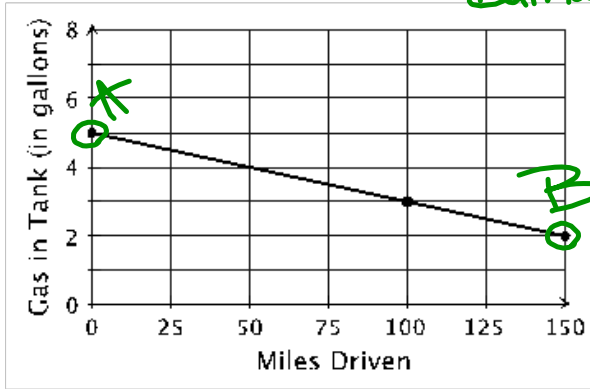
$$m = \frac{4 - 6}{3 - (-8)} = \frac{4 - 6}{3 + 8} = \frac{-2}{11}$$

e. (-3, -5) (-1, -2)

f. (0, 7) (5, 0)

Slope is a measure of **steepness** and **direction**.
Slope describes a **rate of change**.

6. Batman had 5 gallons of gasoline in his motorcycle. After driving 100 miles, he had 3 gallons of gasoline left. The graph below shows ~~Todd's~~ ^{Batman's} situation.



- a. What are the coordinates of two points that you could use to find the slope of the line?

A $(0, 5)$, B $(150, 2)$

$(0, 5)$
 $(150, 2)$

- b. What is the slope of the line? Write in fraction form and **use the units** of measure you find on the y and x axes.

$$m = \frac{\text{Vertical } \Delta}{\text{horizontal } \Delta} = \frac{\Delta \text{ gallons}}{\Delta \text{ miles}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{150 - 0} = \frac{-3 \text{ gallons}}{150 \text{ miles}}$$

rate of change

- c. Write the slope as a *unit rate* that will be in gallons per mile (i.e. gallons per ONE mile). Then write a descriptive sentence explaining HOW you figured out the unit rate.

DIVIDE to figure out how much gas per 1 mile

$$\frac{-3 \text{ gallons}}{150 \text{ miles}} = \boxed{-0.02 \frac{\text{gallons}}{\text{mile}}}$$

unit rate

Tonight's HW

By graph

$$m = \frac{\downarrow 7}{\rightarrow 9} = \boxed{\frac{-7}{9}}$$

(started at point A)

By formula

$$A(-5, 4)$$

$$B(4, -3)$$

$$m = \frac{-3-4}{4-(-5)} = \frac{-3-4}{4+5} = \boxed{\frac{-7}{9}}$$